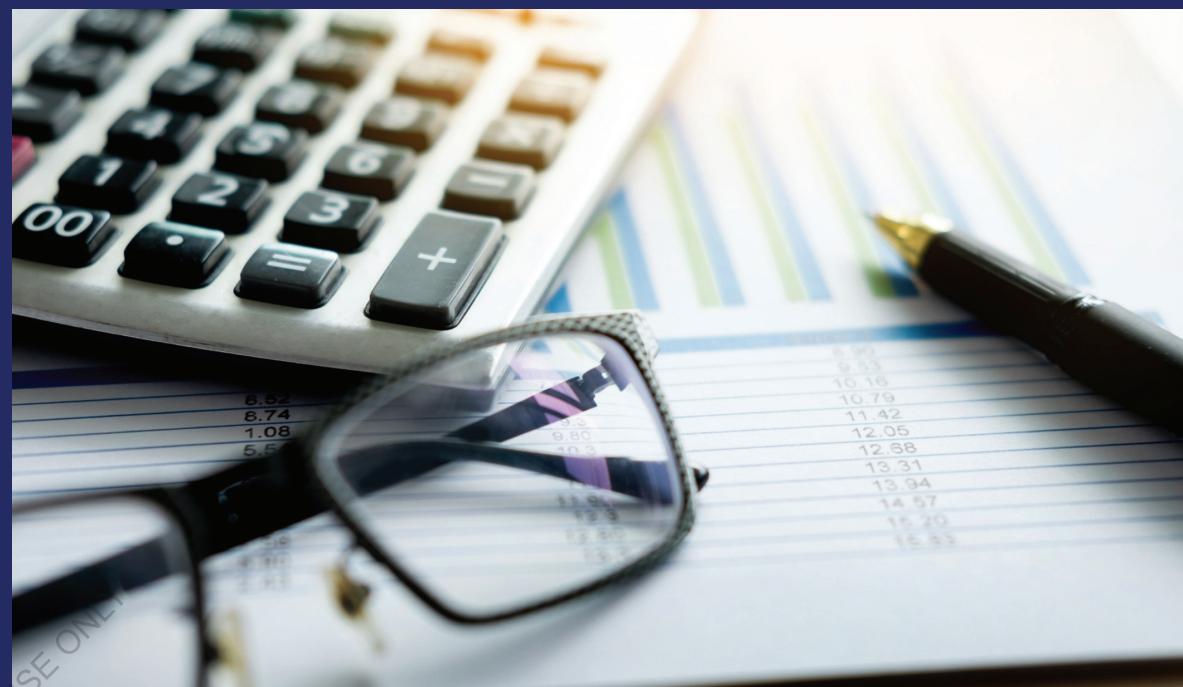


This book provides the latest advances in the theory and practice of Marshall-Olkin family of distributions. In recent years, these distributions have become more widely used in statistical practice, as they allow for the description of intriguing aspects of stochastic models such as non-exchangeability, tail dependencies, and the presence of a unique component. This distribution has been broadly applied to the description of real-world phenomena and the modelling of data in very many disciplines, including insurance, economics, finance, engineering, biology, industry, and medical sciences. This book offers a recent update, essential properties, generalizations, and applications of the Marshall-Olkin family of distributions. It is recommended for researchers working in applied probability and statistics, as well as for practitioners interested in the use of stochastic models in different areas.



Lishamol Tomy
Jiju Gillariose

Statistical Modelling with Marshall-Olkin Generalized Family



Lishamol Tomy, Jiju Gillariose

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List of contributors:

Lishamol Tomy, Ph.D.

Associate Professor

Department of Statistics

Deva Matha College

Kuravilangad, Kerala-686633, India

Email: lishatomy@gmail.com

Jiju Gillariose, Ph.D.

Assistant Professor

Department of Statistics

CHRIST (Deemed to be University)

Hosur Road, Bengaluru-560029, India

Email: jijugillariose@yahoo.com

Meenu Jose

Assistant Professor

Department of Statistics

Carmel College, Mala,

Thrissur, Kerala-680732, India

Email:meenusgc@gmail.com

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