

This book provides the latest advances in the theory and practice of Marshall-Olkin family of distributions. In recent years, these distributions have become more widely used in statistical practice, as they allow for the description of intriguing aspects of stochastic models such as non-exchangeability, tail dependencies, and the presence of a unique component. This distribution has been broadly applied to the description of real-world phenomena and the modelling of data in very many disciplines, including insurance, economics, finance, engineering, biology, industry, and medical sciences. This book offers a recent update, essential properties, generalizations, and applications of the Marshall-Olkin family of distributions. It is recommended for researchers working in applied probability and statistics, as well as for practitioners interested in the use of stochastic models in different areas.



Lishamol Tomy
Jiju Gillariose

- **Lishamol Tomy**, Ph.D., is an Associate Professor in the Department of Statistics, Deva Matha College, Kuravilangad, Kerala, India.
- **Jiju Gillariose**, Ph.D., is an Assistant Professor in the Department of Statistics, Christ (Deemed to be University), Bangalore, Karnataka, India.

Statistical Modelling with Marshall-Olkin Generalized Family

FOR AUTHOR USE ONLY

Lishamol Tomy, Jiju Gillariose



Lishamol Tomy
Jiju Gillariose

Statistical Modelling with Marshall-Olkin Generalized Family

FOR AUTHOR USE ONLY

**Lishamol Tomy
Jiju Gillariose**

**Statistical Modelling with
Marshall-Olkin Generalized
Family**

FOR AUTHOR USE ONLY

LAP LAMBERT Academic Publishing

Imprint

Any brand names and product names mentioned in this book are subject to trademark, brand or patent protection and are trademarks or registered trademarks of their respective holders. The use of brand names, product names, common names, trade names, product descriptions etc. even without a particular marking in this work is in no way to be construed to mean that such names may be regarded as unrestricted in respect of trademark and brand protection legislation and could thus be used by anyone.

Cover image: www.ingimage.com

Publisher:

LAP LAMBERT Academic Publishing

is a trademark of

Dodo Books Indian Ocean Ltd., member of the OmniScriptum S.R.L
Publishing group

str. A.Russo 15, of. 61, Chisinau-2068, Republic of Moldova Europe

Printed at: see last page

ISBN: 978-613-9-46247-6

Copyright © Lishamol Tomy, Jiju Gillariose

Copyright © 2022 Dodo Books Indian Ocean Ltd., member of the
OmniScriptum S.R.L Publishing group

FOR AUTHOR USE ONLY

List of contributors:

Lishamol Tomy, Ph.D.

Associate Professor

Department of Statistics

Deva Matha College

Kuravilangad, Kerala-686633, India

Email: lishatomy@gmail.com

Jiju Gillariose, Ph.D.

Assistant Professor

Department of Statistics

CHRIST (Deemed to be University)

Hosur Road, Bengaluru-560029, India

Email: jijugillariose@yahoo.com

Meenu Jose

Assistant Professor

Department of Statistics

Carmel College, Mala,

Thrissur, Kerala-680732, India

Email: meenusgc@gmail.com

FOR AUTHOR USE ONLY

FOR AUTHOR USE ONLY

Contents

<i>List of Tables</i>	v
<i>List of Figures</i>	vii
1 Marshall-Olkin Generalized Family of Distributions	1
JIJU GILLARIOSE AND LISHAMOL TOMY	
1.1 Introduction	1
1.2 MOE Family of Distributions	2
1.3 Generalized MOE Family	4
1.4 Areas of Application	8
1.4.1 Record Values	8
1.4.2 Entropy Analysis	9
1.4.3 Stress-Strength Reliability modelling	9
1.4.4 Time Series Analysis	10
1.4.5 Acceptance Sampling Inspection Plans	10
Bibliography	11
2 Some Distributions Based on Marshall-Olkin Extended Family	18

JIJU GILLARIOSE AND LISHAMOL TOMY

2.1	Introduction	18
2.2	MOE Power Lomax Distribution	20
2.2.1	Limiting Distributions of Sample Extremes	22
2.2.2	Stress-Strength Analysis and Estimation of Reliability	25
2.2.3	Simulation Study	26
2.2.4	Estimation	28
2.2.5	Data Analysis	29
2.2.6	Truncated Hybrid Double Acceptance Sampling Plan	30
2.2.7	Design of the proposed plan	32
2.2.8	Operating Procedure for the THDASP	32
2.2.9	Minimum Sample Size (n)	33
2.2.10	Operating Characteristics	33
2.2.11	Product Mean Life Ratio $\frac{\lambda}{\lambda_0}$	36
2.2.12	Demonstration of Application to X-Ray Emitting Machine	36
2.3	MOE Modified Lindley Distribution	38
2.3.1	Useful Expansions	39
2.3.2	Moments and Related Quantities	41
2.3.3	Mean deviation (MD), Bonferroni and Lorenz curves	44
2.3.4	Estimation Method with Simulation	44
2.3.5	Estimation Method	44
2.3.6	Applications	46
2.4	Conclusions	53
	References	54
3	On an Inverse Marshall-Olkin Extended Distribution	57
	LISHAMOL TOMY AND JIJU GILLARIOSE	
3.1	Introduction	57

3.2	MOE Inverted Kumaraswamy Distribution	59
3.3	Properties	59
3.3.1	Shape Properties	59
3.3.2	Related Distributions and Transformed Variables	60
3.3.3	Compounding	64
3.3.4	Quantiles and Mode	64
3.3.5	Moments and Moment Generating Function	65
3.3.6	Order Statistics	66
3.3.7	Record Values	67
3.4	Stress-Strength Analysis and Estimation of Reliability	67
3.4.1	Simulation Study	69
3.5	Estimation of Parameters	71
3.6	Application	72
3.7	Conclusions	75
	References	75
4	T- Marshall-Olkin X Family of Distributions	78
	LISHAMOL TOMY AND MEENU JOSE	
4.1	Introduction	78
4.2	T- Marshall-Olkin X Family of Distributions	79
4.3	Some Members of T-Marshall-Olkin X Family of Distributions	82
4.3.1	Exponential- Marshall-Olkin X Family of Distributions	84
4.3.2	Half-Logistic Marshall-Olkin X Family of Distributions	85
4.4	Properties of HLMO-X family of distribution	88
4.4.1	Some Useful Expansions	88
4.4.2	Moments, Generating Functions And Mean Deviation	89
4.4.3	Order Statistics	91
4.4.4	Asymptotic Distributions of Sample Extremes	92

4.5	Half Logistic- Marshall-Olkin Lomax Distribution	94
4.5.1	Linear Representation	97
4.5.2	Moments and Generating Functions	98
4.5.3	Order Statistics	100
4.5.4	Asymptotic Distributions of Sample Extremes	100
4.6	Estimation of parameters by maximum likelihood method	102
4.6.1	Simulation Study	103
4.7	Applications	104
4.7.1	The Tobacco Data Sets	104
4.7.2	Service Times of Aircraft Windshield Datasets	107
4.8	Conclusions	112
	References	112

FOR AUTHOR USE ONLY

List of Tables

2.1	Numerical values of AB and AMSE when $\beta = 9, \theta = 4$ and $\lambda = 3$	27
2.2	Numerical values of ACL and CP when $\beta = 9, \theta = 4$ and $\lambda = 3$	28
2.3	Comparison criterion for the data set	31
2.4	Developed minimum sample size for MOEPL distribution and the corresponding acceptance number c when $c = 0.2, \beta = 0.5$ and $\theta = 0.3$	34
2.5	Design parameters of the THDASP for the MOEPL distribution with $c = 0.2, \beta = 0.5$ and $\theta = 0.3$	35
2.6	Minimum ratio of true mean life to specified mean for acceptance of lot of when the lifetime of a product follows a MOEPL distribution.	37
2.7	Probability of Acceptance values for X-ray Emission Machine Data for MOEPL distribution	38
2.8	Some characteristics of the of the MOEML distribution	42
2.9	Comparison criterion for Data set 1	51
2.10	Comparison criterion for Data set 2	51
3.1	Transformations Applied to MOEIKum and the Resulting Distributions	63

3.2	Numerical values of AB and AMSE when $\beta = 0.5$ and $\lambda = 0.9$	70
3.3	Numerical values of ACL and CP when $\beta = 0.5$ and $\lambda = 0.9$	71
3.4	Comparison criterion for the data set	73
4.1	Some members of T Marshall-Olkin X family of distributions for different T distributions	83
4.2	μ_1, μ_2, μ_3, S and K for various choices of parameters	101
4.3	AEs, Bias and MSE of parameters based on 1000 simulations of the HLMOL distribution	105
4.4	Table 3:(Continued)	106
4.5	Estimated values, log-likelihood, AIC, BIC, AICc, A* and W* for the tobacco data set	108
4.6	Estimated values, log-likelihood, AIC, BIC, AICc, A* and W* for the service times of aircraft windshield data set	110

List of Figures

2.1	Plots of the PDF of the MOEPL model	21
2.2	Plots of the HRF of the MOEPL model	21
2.3	P-P plot for X-Ray emitting machine that assumes MOEPL distribution	38
2.4	Plots of the PDF of the MOEML distribution	40
2.5	Plots of the HRF of the MOEML distribution	40
2.6	Plots of the Bias of MLE for $\eta = 0.5$ and $c = 3$	46
2.7	Plots of the MSE of MLE for $\eta = 0.5$ and $c = 3$	47
2.8	Plots of the Bias of MLE for $\eta = 2$ and $c = 3$	47
2.9	Plots of the MSE of MLE for $\eta = 2$ and $c = 3$	48
2.10	Plots of the Bias of MLE for $\eta = 0.5$ and $c = 0.5$	48
2.11	Plots of the MSE of MLE for $\eta = 0.5$ and $c = 0.5$	49
2.12	Plots of the Bias of MLE for $\eta = 3$ and $c = 0.5$	49
2.13	Plots of the MSE of MLE for $\eta = 3$ and $c = 0.5$	50
2.14	PP and QQ plots of MOEML for Dataset 1	52
2.15	PP and QQ plots of MOEML for Dataset 2	52

3.1	Plots of the PDF of the MOEIKum model	60
3.2	Plot of the HRF of the MOEIKum model	61
3.3	Probability plots for the data	74
3.4	Fitted PDFs and the observed histogram for the data set	76
4.1	PDF of HLMOL for Various Values of α, θ, λ and c	95
4.2	HRF of HLMOL for Various Values of α, θ, λ and c	96
4.3	Estimated PDF and CDF for the Kw-GL,WL,GL,PL,EL,HLMOL distributions for tobacco data sets.	109
4.4	Estimated PDF and CDF for the Kw-GL,WL,GL,PL,EL,HLMOL distributions for service times of aircraft windshield data sets.	111

FOR AUTHOR USE ONLY